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Elementary Mathematics

25/11/2025

MTH 101

Lecturer: Dr. Mrs. T.E Efor

Topic: Real Sequences, Series + Polynomials

* **Natural Numbers (N):** Are Counting numbers. They are positive ~~whole~~ whole numbers. Natural Numbers is denoted by N.

$$N = \{1, 2, 3 \ldots \infty\} \rightarrow \text{Infinite numbers}$$

$$D = \{1, 2, 3 \ldots 10\} \rightarrow \text{Finite numbers}$$

Elements or Entries.

Properties of Natural Numbers

i) Natural Numbers are closed under addition [closure Law)
$\forall$ (for all) $m, n \in N$.

ii) Natural Numbers are closed under multiplication
$\forall m, n \in N, mn \in N$

iii) Natural Numbers are not closed under subtraction and division.

* **Integers (Z):** Is the set of negative or positive whole numbers.

$$Z = \{-2, -1, 0, 1, 2, 3\}$$

Properties of Integers

i) It is closed under addition

ii) It is closed under Subtraction

iii) It is closed under multiplication

i.e $\forall m, n \in Z, mn \in Z$

Rational Numbers: These are numbers in form of $\frac{m}{n}$ is the largest term.

$\frac{m}{n}$ Such that the denominator n should not be equal to 0 ie $n \neq 0$

Irrational Numbers: Irrational Numbers are denoted as $\mathbb{IQ}$ or Q'.

REAL NUMBERS:

Denoted by R

$R = Q \cup Q'$

Real numbers are closed in addition, Subtraction, multiplication and division.

SEQUENCE:

A Sequence is a function $f: D \overset{\text{subset}}{\subseteq} S - S$

$f: D \subseteq N - R$ → Real Sequence

Exercise:

Arrange the following in ascending order

$$\underset{4}{R}, \underset{3}{Q}, \underset{1}{N}, \underset{2}{Z}$$

Ans $N \subseteq Z \subseteq Q \subseteq R$.

A Sequence in a Set S is a function from subset D of the natural numbers (N)

Mathematically, A Sequence f in S

$= f : D \subseteq N \rightarrow S$

Real Sequence $= f : D \subseteq N \rightarrow R$

* Real Sequence is a Sequence in which $S = R$

Thus, a real Sequence $\{a_n\}_n \subseteq D \subseteq N$. It is a Function

$f : D \in N \rightarrow R$

Such that

$f_N = a_n \in R \; \forall N \in D \subseteq N$

Where $n = 1$

$1, f_n = a_1, 2, f_n = a_2, 3, f_n = a_3$

$f_{n-1} = a_{n-1}$

Topic 3

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Polynomials

Polynomial is a function (f) of an Independent Variable (x) Represented as:

$f(x) =$

$a_n x^n + a_{n-1} x + \dots + a_2 x^2 + ax + a$

$a_0, a_1, a_2, \dots a_n, a_{n-1}$ are real constant

Example:

- A cubic equation
- " linear "
- " quadratic "
- " quartic "

$f(x) = 3x^2 + 6x + 10$

$f(x) = 6x^2 + 4x + 2$

$f(x) = 2x + 5$

* Degree or order of a polynomial is the highest power of the variable in the polynomial.

Rational Functions

A rational function is the ration of 2 polynomials of variable x, represented by

$$R(x) = \frac{P(x)}{Q(x)}, \text{ where}$$

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a x + a_0$$

and

$$Q(x) = b_n x^n + b_n x^n + b_{n-1} x^{n-1} + \dots + b_2 x^2 + b_1 x + b_0 \text{ and } Q(x) \neq 0$$

$$x \in D(Rx)$$

<u>Example</u>

$$R(x) = \frac{3x^2 + 6x + 10}{2x + 5}$$

When $2x + 5 = 0$

$= 2x + 5 - 5 = -5$

$= 2x = -5$

$= x = -\frac{5}{2}$

* When you divide 2 polynomials and the answer is a polynomial; It is a Factor.

All this points along the horizontal axis where the factor are found along the horizontal axis are called roots of the polynomial.

Exercise:

$f(x) = x^4 - 8x^3 + 11x^2 + 8x - 12$

Find the factor of $f(x)$

-12: $1, 12, 2, -6, 3, -4, -1, 12, -2, 6, -3, 4$
(are all factors of -12).

A) $F(1) = (1)^4 - 8(1)^3 + 11(1)^2 + 8(1) - 12 = 0$
B) $f(2) = (2)^4 - 8(2)^3 + 11(2)^2 + 8(2) - 12 = 0$
C) $f(3) = (3)^4 - 8(3)^3 + 11(3)^2 + 8(3) - 12 = -24$
D) $f(4) = (4)^4 - 8(4)^3 + 11(4)^2 + 8(4) - 12 = -60$
E) $f(6) = (6)^4 - 8(6)^3 + 11(6)^2 + 8(6) - 12 = 0$
F) $f(12) = (12)^4 - 8(12)^3 + 11(12)^2 + 8(12) - 12 = 8580$

$\therefore$

When $x = -1$ $(x+1) = 0$
When $x = 1$, $(x-1) = 0$
When $x = 2$, $(x-2) = 0$
When $x = 3$, $(x-3) = 0$
$\therefore$ These are Factors

Factor theorem:

It states that $f(x)$ is divisible by $(ax+b)$ if and only if $f\left(\frac{-b}{a}\right) = 0$, and $\frac{-b}{a}$ is the root of $f(x)$.

Remainder theorem:

It states that if $f(x)$ is divided by $(ax+b)$, the remainder $f\left(\frac{-b}{a}\right)$.

Dr Sir Law Adam MTH 101 02/12/2025

Indices

* Laws of Indices: *

1. $x^m \times x^n = x^{m+n}$
2. $\frac{x^m}{x^n} = x^{m-n}$, $x^n \neq 0$
3. $x^1 = 1$
4. $x^{-n} = \frac{1}{x^n}$, $n \in Z$
5. $x^{\frac{m}{n}} = (\sqrt[n]{x})^m$
6. $x^{mn} = (x^n)^n$
7. $(xy)^m = x^m y^m$

Solving Problems Involving Indices

1) Solve for $\frac{\sqrt{ab} \times a^{\frac{1}{3}} \times 2b^{1/4}}{(a^{10}b^9)^{1/2}}$

Solution

$$\sqrt{ab} \times a^{1/3} \times 2b^{1/4} = \frac{a^{1/2} \times b^{1/2} \times a^{1/3} \times 2b^{1/4}}{a^{1/2} \times b}$$

$$= \frac{2\cancel{a^{5/6}}\,\cancel{b^{3/4}}}{\cancel{a^{5/6}}\,\cancel{b^{3/4}}} = 2 \,//$$

2) $2^{2y} - 5(2^y) + 4 = 0$

Solution

$2^{2y} - 5(2^y) 4 = 0$

Let $2^y = x$

$x^2 - 5x + 4 = 0$

$(x-1)(x-4) = 0$

$x = 1 = 0$ or $x - 4 = 0$

$x = 1$ or 4

$2^y = 1$ or $2^y = 4$

$\cancel{2}^y = \cancel{2}^0$ or $\cancel{2}^y = \cancel{2}^2$

$y = 0$ or $y = 2$

Logarithm

$\log_B A = x$

$B^x = A$

Eg $\log_{10} 4 = x$

$10^x = A$

* Laws of Logarithm *

1. $\log_a^{mn} = \log_a^m + \log_a^n$

2. $\log \frac{m}{n} = \log \frac{m}{a} - \log_a^n$

3. ~~log~~ $\log_b a^n = n\log_b^a$

4. $\log_b^a = \frac{1}{\log_a^b}$

5. $\log_b^a =$

6. $\log^1 \alpha = 0$

Topic 3

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Complex Numbers, Circular Measure & Trigonometric Ratios

$$\mathbb{N} \subset \mathbb{Z} \subset (\mathbb{Q} \cup \mathbb{I}\mathbb{Q}) \subset \mathbb{R} \subset \mathbb{C}$$

The set of Natural numbers $\mathbb{N}$	The set of Integers $\mathbb{Z}$	The set of rational numbers	The set of Irrational nos.	The real number system	The set of Complex numbers

This is designated as $\mathbb{N}$ or N

* $\mathbb{N} = \{1, 2, 3 \ldots\ldots, n \ldots\}$ (Set of Natural Numbers)

Axioms of $\mathbb{N}$

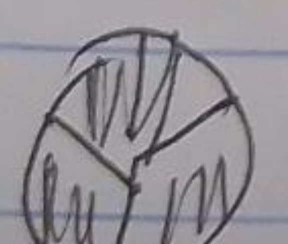
Let $n, m, \in \mathbb{N}$

i) $n + m \in N, \forall n, m$

ii) $nm \in N, \forall n, m$

* $\mathbb{Z} = \{\ldots, \underbrace{-3, -2, -1}_{\text{Negative } \mathbb{Z}}, 0, \underbrace{1, 2, 3 \ldots}_{\text{Positive } \mathbb{Z} \text{ or } \mathbb{N}}\}$ (Set of ~~Rational Numbers~~ Integers)

$\mathbb{Q} = \{\frac{p}{q} \mid p, q \in \mathbb{Z}, \text{ provided } q \neq 0\}$ (Set of Rational numbers).



$= \frac{1}{3} + \frac{2}{3} = \frac{1+2}{3} = \frac{3}{3} = 1$ $\therefore$ Rational numbers are placed as fraction e.g $\frac{a}{b}$

$\frac{1}{3} = 1:3$ The ratio of 1:3 (Rational)

NB: Not all numbers in fraction are rational numbers.

$$\left.\begin{aligned}
\tfrac{1}{2} &= 0.5 \\
\tfrac{1}{3} &= 0.333\ldots \\
\tfrac{1}{4} &= 0.25 \\
\tfrac{1}{5} &= 0.2 \\
\tfrac{1}{6} &= 0.16666 \\
\tfrac{1}{7} &= 0.142857142857\ldots \\
\tfrac{1}{8} &= 0.125 \\
\tfrac{1}{9} &= 0.1111111\ldots
\end{aligned}\right\}$$

* Numbers in decimal or repeated floating decimal can be converted as Rational numbers.

* $IQ = \{\underbrace{\sqrt{2}, \sqrt{13}, \sqrt{111}, \sqrt{\text{prime no}}, \sqrt{\text{Non-Perfect square}}}_{\text{Surd}}, \pi, e \ldots$
(Set of Irrational Numbers)

$$\pi = 3.14|1592\,15$$

$$\frac{22}{7} = 3.14|2857142157\ldots$$

NB: π is not equal to $\frac{22}{7}$ but taken in approximatio of $\frac{22}{7}$.

Exponential, $e = 2.918281828459045235\ldots$

Or $e = \lim_{m \to \infty} \left(1 + \frac{1}{n}\right)^n = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^{n+1}$

$\boxed{\log_e a = \ln a}$ → logarithm to the base of the Exponential, also called Natural logarithm

NB: Surds are Irrational numbers in a root form.

* To make an Irrational number a rational number [Rationalization]

Eg1: $\sqrt{5}$

Solution

$$\sqrt{5} \times \sqrt{5} = 5^{1/2} \cdot 5^{1/2}$$
$$= 5^{1/2 + 1/2}$$
$$= 5^1 = 5.$$

Eg 2: $\frac{1}{\sqrt{5}}$

Solution

$$\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \times 1 = \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$
$$= \frac{\sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\therefore \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

* $\mathbb{Q}' \cup \mathbb{Q} = \mathbb{R}$ (set of Real numbers)

e.g $x^2 + 1 = 0 \Rightarrow x^2 = -1$

$\therefore x = \sqrt{-1}$

* $\mathbb{C} = \{z : z = a + bi,\ a, b \in \mathbb{R},\ i = \sqrt{-1}\}$

Or

$\mathbb{C} = \{a + bi \mid a, b \in \mathbb{R},\ i = \sqrt{-1}\}$

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~~Partical Fractions~~

Partial Fractions

$$\frac{1}{2} + \frac{1}{3} = \frac{3+2}{6} = \frac{5}{6}$$

Eg. 1 $\dfrac{3x - 1}{(x3')(x-3)} = \dfrac{A}{(x+1)} + \dfrac{B}{(x-1)}$

$c(A + B) \Rightarrow 3x - 1 = A(x+1)(x-1) + \frac{B}{x}(x+1)(x-1)$

MTH 101

Topic:

PARTIAL FRACTION

10/12/2025

A Partial fraction is a natural Function that can be written as a sum of 2 or more Similar rational Formular.

$$\frac{1}{2} + \frac{1}{3} = \frac{3+2}{6} = \frac{5}{6}$$

Eg 1: $\frac{3x-1}{(x+1(x-3)}$

Solution

$$\frac{3x-1}{(x+1)(x-3)}$$

$$= \frac{A}{(x+1)} + \frac{B}{(x-3)}$$

$$C(A+B) \Rightarrow 3x-1 = \frac{A(x+1)(x-3)}{(x+1)} = \frac{B(x+1(x-3)}{(x-3)}$$

Multiply through the L.C.M $(x+1)(x-3)$ to give

$$3x-1 = A(x-3) + B(x+1)$$

Eliminate A or B by ~~multiply~~ Inputing the root values of the Factors $x+1$, $x-3$

When $x-3=0$, $x=3$

$$\Rightarrow 3(3)-1 = A(0) + B(3+1)$$

$$\frac{8}{4} = \frac{4B}{4} \quad \therefore B = 2$$

When $x = -1$

$3(-1) - 1 = A(-1-3) + B(0)$

$\frac{\cancel{-4}}{\cancel{-4}} = \frac{\cancel{-4}A}{\cancel{-4}}$

$\therefore A = 1$

$\therefore$ The partial fraction is

$$\frac{3x-1}{(x+1)(x-3)} = \frac{1}{x+1} + \frac{2}{x-3}$$

Ans

Eg 2: $\frac{x^2+1}{(x+1)(x-2)}$

Solution

$$\frac{x^2+1}{(x+1)(x-2)}$$

$$= \frac{Ax+B}{x+1} + \frac{C}{x-2}$$

Multiply by the denominator and expand.

$$x^2+1 = \frac{Ax+B\cancel{(x+1)}(x-2)}{x+1} + \frac{C(x+1)\cancel{(x-2)}}{x-2}$$

$x^2+1 = Ax^2 + 2Ax + Bx - 2B + Cx + C$

$x^2+1 = Ax^2 - 2Ax + Bx - 2B + Cx + C$

CLT (Collect like terms)

$$x^2 + 1 = Ax^2 - 2Ax + Bx - 2B + cx + c$$

$$x^2 + 1 \Rightarrow Ax^2 + x(-2A + B + c) - 2B + c$$

$$x^2 + 1 \Rightarrow Ax^2 - x(-2A + B + c) - 2B + c$$

Equate Co-efficients

$$1x^2 = Ax^2 \quad \therefore A = 1$$

But $A = 1$

$$-2A + B + C = 0$$

$$-2 + B + C = 0$$

$$\Rightarrow B + C = 2 \quad \cdots (1)$$

But $-2B + c = 1 \quad \cdots (2)$

Solving from eqn (1) & (2) B and c Simultaneously

$$B + C = 2$$

$$\underline{-2B + C = 1}$$

$$-3B = -1 \quad \Rightarrow \frac{B}{-3} = \frac{+1}{+3} \quad \therefore B = \frac{1}{3}$$

Substitute $B = \frac{1}{3}$ in eqn (1)

$$\frac{1}{3} + c = 2$$

$$C = 2 - \frac{1}{3} = \frac{6-1}{3}$$

$$C = \frac{5}{3}$$

$A = 1$, $B = \frac{1}{3}$, $C = \frac{5}{3}$, $B + C = 2$

Solve for A in eqn (3) to get

$B + C = 2 \quad \ldots\ldots (4)$

$\therefore$ The Partial fraction is :

$$\frac{x^2+1}{(x+1)(x-2)} = \frac{x+\frac{1}{3}}{x+1} = \frac{5/3}{x-2}$$

$$\frac{x^2+1}{(x+1)(x-2)} = \frac{3x+1}{3(x+1)} + \frac{5}{3(x-2)}$$

Eg. 3 : $\dfrac{2x+3}{(x+2)(x^2+1)}$

Solution

$$\frac{2x+3}{(x+2)(x^2+1)}$$

$$= \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$$2x+3 = A(x^2+1) + (2x+c)(x+2)$$

$$= Ax^2 + A + Bx^2 + 2Bx + (x+2c)$$

$$= (A+B)x^2 + (2B+C)x + (A+2C)$$

By Comparison of both Sides

$A + B = 0 \quad \ldots\ldots (1)$

$$2B + C = 2 \quad \ldots\ldots (2)$$
$$A + 2C = 3 \quad \ldots\ldots (3)$$

from eqn (1) $A = B$

Substitute in eqn (3) B For A

$$-B + 2C = 3 \quad \ldots\ldots (4)$$

Solving eqn (2) & (4) Simultaneously we get

$$2B + C = 2$$
$$\underline{+2(-B + 2C) = 2(3)}$$ [multiply eqn (4) by (2) and add eqn (2)]
$$0 + 5C = 8$$

$$C = \frac{8}{5}$$

Substitute $\frac{8}{5}$ For C in eqn (3) to get A

$$A + 2\left(\frac{8}{5}\right) = 3$$

$$A = 3 - \frac{16}{5} = \frac{15 - 16}{5} = \frac{-1}{5}$$

But $A = -B$ from eqn (1)

$$\therefore B = -(-1/5) = \frac{1}{5}$$

$$\therefore A = -1/5, \; B = 1/5, \; C = 8/5$$

$$\therefore \frac{2x+5}{(x+2)(x^2+1)} = \frac{-1}{5(x+2)} + \frac{\frac{1}{5}x + \frac{8}{5}}{x^2+1}$$

$$= \frac{x+8}{5(x^2+1)} = \frac{1}{5(x+2)} \text{ Ans.}$$

Eg. 4: $\frac{2x+5}{(x+4)^2(x-2)}$ $\cancel{= A}$

Solution

$$\frac{2x+5}{(x+4)^2(x-2)}$$

$$= \frac{A}{(x+4)^2} + \frac{B}{(x+4)} + \frac{C}{(x-2)}$$

NB: We solve this way because $(x+4)^2 = (x+4)(x+4)$ and since they are the same the constants would be the sa

Multiply through the denominator

$\Rightarrow 2x+5 = A(x-2) + B(x+4)(x-2) + C(x+4)^2$

When $x = 2$ from $(x-2) = 0$

$\Rightarrow 2(2) + 5 = A(0) + B(0) + C(6)^2$

$\Rightarrow 9 = 36C$

$\therefore C = \frac{9}{36}$

$C = \frac{1}{4}$

When $x = -4$ from $x + 4 = 0$

$\Rightarrow 2(-4) + 5 = A(-4-2) + B(0) + C(0)$

$\Rightarrow -3 = -6A$

$\therefore A = \frac{-3}{-6}, \quad A = \frac{1}{2}$

Expand the equation to be able to solve for B

$\Rightarrow 2x + 5 = Ax - 2A + Bx^2 + 2Bx - 8B + (x^2+8)(x+16c)$

Collect like terms

$2x + 5 = Bx^2 + Cx^2 + Ax + 2Bx + 8Cx - 2A - 8B + 16C$

Factorize

$2x + 5 = x^2(B + C) + x(A + 2B + 8C) - 2A - 8B + 16C$

Equating the Co-efficient of x^2

$0 = B + C \Rightarrow B = -C$

But $C = \frac{1}{4}$ $\quad \therefore B = \frac{-1}{4}$

$\therefore$ The partial fraction, Substituting A, B & C, is

$$\frac{2x+5}{(x+4)^2(x-2)} = \frac{1}{2(x+4)^2} - \frac{1}{4(x+4)} + \frac{1}{4(x-2)} \text{ Ans}$$